

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 1	Daily Tutorial Sheet - 4
------------------	---------------------------------

46.(C) Consider: $\cos 2\alpha = \frac{3 \cos 2\beta - 1}{3 - \cos 2\beta}$

Apply 'C' and 'D', $\frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{3 - \cos 2\beta - 3 \cos 2\beta + 1}{3 - \cos 2\beta + 3 \cos 2\beta - 1}$

$$\Rightarrow \frac{2 \sin^2 \alpha}{2 \cos^2 \alpha} = \frac{4 - 4 \cos 2\beta}{2 + 2 \cos 2\beta} \Rightarrow \tan^2 \alpha = \frac{4(1 - \cos 2\beta)}{2(1 + \cos 2\beta)} \Rightarrow \tan^2 \alpha = 2 \tan^2 \beta \Rightarrow \tan \alpha = \sqrt{2} \tan \beta$$

47.(AB) Consider: $a \cos \theta + b \sin \theta = c$

$$\Rightarrow a \cos \theta = c - b \sin \theta \Rightarrow a^2 \cos^2 \theta = (c - b \sin \theta)^2 \text{ (Squaring both sides)}$$

$$\Rightarrow a^2 (1 - \sin^2 \theta) = c^2 - 2bc \sin \theta + b^2 \sin^2 \theta \Rightarrow (a^2 + b^2) \sin^2 \theta - 2bc \sin \theta + c^2 - a^2 = 0$$

As α and β are values of ' θ ' as given:

$\therefore$ Roots of above equation are $\sin \alpha$ and $\sin \beta$.

$$\Rightarrow \sin \alpha + \sin \beta = \text{sum of roots} = \frac{2bc}{a^2 + b^2} \text{ and } \sin \alpha \cdot \sin \beta = \text{product of roots} = \frac{c^2 - a^2}{a^2 + b^2}$$

48.(A) $\sin (y + z - x)$, $\sin (z + x - y)$, and $\sin (x + y - z)$ are in A.P.

$$\Rightarrow \sin (z + x - y) - \sin (y + z - x) = \sin (x + y - z) - \sin (z + x - y)$$

$$\Rightarrow 2 \cos z \cdot \sin (x - y) = 2 \cos x \cdot \sin (y - z)$$

[By using $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$]

Dividing both sides by $2 \cos x \cdot \cos y \cdot \cos z$, to get

$$\frac{\sin (x - y)}{\cos x \cdot \cos y} = \frac{\sin (y - z)}{\cos y \cdot \cos z} \Rightarrow \frac{\sin x \cdot \cos y - \cos x \cdot \sin y}{\cos x \cdot \cos y} = \frac{\sin y \cos z - \sin z \cdot \cos y}{\cos y \cdot \cos z}$$

$$\Rightarrow \tan x - \tan y = \tan y - \tan z \Rightarrow 2 \tan y = \tan x + \tan z$$

$\Rightarrow \tan x, \tan y, \tan z$ are also in A.P.

49.(B) As 1 radian = 57° approx. and $\sin 57^\circ > \sin 1^\circ \Rightarrow \sin 1^\circ < \sin 1$

50.(BC) As $\cos (\theta - \phi)$, $\cos \theta$, $\cos (\theta + \phi)$ are in H.P

$$\Rightarrow \cos \theta = \frac{2 \cos (\theta - \phi) \cdot \cos (\theta + \phi)}{\cos (\theta + \phi) + \cos (\theta - \phi)} = \frac{2 (\cos^2 \theta - \sin^2 \phi)}{2 \cos \theta \cdot \cos \phi} \text{ [If } a, b, c \text{ are in H.P. then } b = \frac{2ac}{a+c} \text{]}$$

$$\Rightarrow \cos^2 \theta \cdot \cos \phi = \cos^2 \theta - \sin^2 \phi \Rightarrow \sin^2 \phi = \cos^2 \theta (1 - \cos \phi)$$

$$\Rightarrow \cos^2 \theta = \frac{\sin^2 \phi}{1 - \cos \phi} = \frac{4 \sin^2 \frac{\phi}{2} \cdot \cos^2 \frac{\phi}{2}}{2 \sin^2 \frac{\phi}{2}}$$

$$\Rightarrow \cos^2 \theta = 2 \cos^2 \frac{\phi}{2} \Rightarrow \cos^2 \theta \cdot \sec^2 \frac{\phi}{2} = 2 \Rightarrow \cos \theta \cdot \sec \frac{\phi}{2} = \pm \sqrt{2}$$

51.(B) Consider: $2 \sin^2 \theta + 2 \cos (\theta + \alpha) \cdot 2 \sin \alpha \cdot \sin \theta + \cos 2 (\alpha + \theta)$

$$\Rightarrow 2 \sin^2 \theta + 2 \cos (\theta + \alpha) [\cos (\theta - \alpha) - \cos (\theta + \alpha)] + \cos 2 (\theta + \alpha)$$

[By using $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$]

$$\Rightarrow 2 \sin^2 \theta + 2 \cos (\theta + \alpha) \cdot \cos (\theta - \alpha) - 2 \cos^2 (\theta + \alpha) + \cos 2 (\theta + \alpha)$$

$$\Rightarrow 2 \sin^2 \theta + 2 (\cos^2 \theta - \sin^2 \alpha) - 2 \cos^2 (\theta + \alpha) + 2 \cos^2 (\alpha + \theta) - 1$$

$$\Rightarrow 2 \sin^2 \theta + 2 \cos^2 \theta - 2 \sin^2 \alpha - 1$$

$$\Rightarrow 2 (\sin^2 \theta + \cos^2 \theta) - 2 \sin^2 \alpha - 1 \Rightarrow 1 - 2 \sin^2 \alpha = \cos 2 \alpha$$

Which is independent of ' θ '.

52.(D) We have, $\sin^6 \theta + \cos^6 \theta + k \cos^2 2\theta = 1$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta) (\sin^4 \theta - \sin^2 \theta \cdot \cos^2 \theta + \cos^4 \theta) + k \cos^2 2\theta = 1$$

$$\Rightarrow \sin^4 \theta + \cos^4 \theta - \sin^2 \theta \cdot \cos^2 \theta + k \cos^2 2\theta = 1$$

$$\Rightarrow (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cdot \cos^2 \theta + k \cos^2 2\theta = 1$$

$$\Rightarrow 3 \sin^2 \theta \cdot \cos^2 \theta = k \cos^2 2\theta \Rightarrow \frac{3}{4} \sin^2 2\theta = k \cos^2 2\theta \Rightarrow k = \frac{3}{4} \tan^2 2\theta$$

53.(C) As: $\sin 15^\circ \cdot \cos 15^\circ \Rightarrow \frac{1}{2} \sin 30^\circ = \frac{1}{4}$ which is rational no.

54.(D) Since $\sin \theta_1 + \sin \theta_2 + \sin \theta_3 = 3 \Rightarrow \sin \theta_1 = \sin \theta_2 = \sin \theta_3 = 1$
 $\therefore \cos \theta_1 = \cos \theta_2 = \cos \theta_3 = 0 \Rightarrow \cos \theta_1 + \cos \theta_2 + \cos \theta_3 = 0$

55.(AC) Consider: $\sin 2x + \sin 4x = 2 \sin 3x \Rightarrow (\sin 2x + \sin 4x) - 2 \sin 3x = 0$

$$\Rightarrow 2 \sin 3x \cdot \cos x - 2 \sin 3x = 0 \Rightarrow 2 \sin 3x (\cos x - 1) = 0$$

$$\Rightarrow \text{Either } \sin 3x = 0 \quad \text{or} \quad \cos x = 1$$

$$\Rightarrow 3x = n\pi \quad \text{or} \quad x = 2m\pi$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = 2m\pi \quad [\text{where } n \in I]$$

56.(C) Consider : $8 \tan^2 \frac{x}{2} = 1 + \sec x \Rightarrow 8 \left(\frac{1 - \cos x}{1 + \cos x} \right) = 1 + \frac{1}{\cos x} \Rightarrow 8 \cos x - 8 \cos^2 x = (1 + \cos x)^2$

$$\Rightarrow 8 \cos x - 8 \cos^2 x = 1 + \cos^2 x + 2 \cos x \Rightarrow 9 \cos^2 x - 6 \cos x + 1 = 0 \Rightarrow (3 \cos x - 1)^2 = 0$$

$$\Rightarrow \cos x = \frac{1}{3} = \cos \alpha \Rightarrow x = 2n\pi \pm \alpha \quad \text{where } \alpha = \cos^{-1} \frac{1}{3} \Rightarrow x = 2n\pi \pm \cos^{-1} \left(\frac{1}{3} \right)$$

57.(C) The given equation is meaningful when $\cos x = 0$ i.e. $\sin x \neq 1$ or -1

Consider : $\tan x + \sec x = 2 \cos x$

$$\Rightarrow \sin x + 1 = 2 \cos^2 x = 2 (1 - \sin^2 x) \Rightarrow 2 \sin^2 x + \sin x - 1 = 0 \Rightarrow (1 + \sin x) (2 \sin x - 1) = 0$$

$$\Rightarrow \sin x = \frac{1}{2} \text{ and } \sin x + 1 \neq 0 \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ in } [0, 2\pi] \text{ Hence no. of solutions is 2.}$$

58.(BC) We have, $4 \sin^4 x + \cos^4 x = 1 \Rightarrow 4 \sin^4 x = 1 - \cos^4 x = (1 - \cos^2 x)(1 + \cos^2 x)$

$$\Rightarrow 4 \sin^4 x - [(1 - \cos^2 x)(1 + \cos^2 x)] = 0 \Rightarrow 4 \sin^4 x - [\sin^2 x(1 + \cos^2 x)] = 0$$

$$\Rightarrow \sin^2 x [4 \sin^2 x - (1 + \cos^2 x)] = 0 \Rightarrow \sin^2 x [4 \sin^2 x - [1 + (1 - \sin^2 x)]] = 0$$

$$\Rightarrow \sin^2 x [4 \sin^2 x - 1 - (1 - \sin^2 x)] = 0 \Rightarrow \sin^2 x (5 \sin^2 x - 2) = 0$$

$$\text{Either } \sin x = 0 \text{ or } \sin x = \pm \sqrt{\frac{2}{5}} \Rightarrow x = n\pi \text{ or } x = n\pi \pm \alpha \text{ where } \alpha = \sin^{-1} \sqrt{\frac{2}{5}} \text{ and } n \in I$$

59.(A) We know that in a triangle $A + B + C = 180^\circ$ and $2B = A + C$ (As A, B, C are in A.P.)

$$\Rightarrow 3B = 180 \text{ or } B = 60^\circ$$

Now ' θ ' be the common difference between A, B and C . Then $C - A = 2\theta$... (i)

$$\Rightarrow \sin(C - A) = \frac{1}{2} \text{ (given)} \Rightarrow \sin(2\theta) = \frac{1}{2} \quad [\text{using (i)}]$$

$$\Rightarrow 2\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{12} \text{ or } \frac{5\pi}{12}$$

$$\text{Now: } A = B - \theta \Rightarrow \frac{\pi}{3} - \frac{\pi}{12} \text{ or } \frac{\pi}{3} - \frac{5\pi}{12}$$

$$\Rightarrow A = \frac{\pi}{4} \text{ as } A \text{ cannot be less than '0' and } C = \frac{\pi}{3} + \frac{\pi}{12} - \frac{5\pi}{12}$$

$$\Rightarrow A = 45^\circ, B = 60^\circ, C = 75^\circ$$

60.(B) Consider : $\sum_{i=1}^n \cos \theta_i = n \Rightarrow \cos \theta_1 + \cos \theta_2 + \cos \theta_3 + \dots + \cos \theta_n = \underbrace{1 + 1 + 1 \dots + 1}_{n \text{ times}}$

which is valid only when $\cos \theta_1 = 1, \cos \theta_2 = 1, \cos \theta_3 = 1, \dots, \cos \theta_n = 1$

$$\Rightarrow \sin \theta_1 = \sin \theta_2 = \sin \theta_3 = \dots = \sin \theta_n = 0 \Rightarrow \sum_{i=1}^n \sin \theta_i = 0$$